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A Study of Transient Concentration Distribution of Pollutions Emitted from Area Source

D.V.S. Kushwah, V.P. Saxena* and Vandana Kushwah

Institute of Information Technology and Management, Gwalior
** Ex-Vice Chancellor, Jiwaji University, Gwalior (MP) INDIA*

ABSTRACT : The purpose of this work is to develop an analytical model for atmospheric distribution and transport of pollutants emitted from area source. However, the concentration distribution of pollutants is assumed to unsteady. One dimensional unsteady state model includes the chemical reaction of the pollutant which is considered to vary with vertical height through troposphere. Hence, the finite element method would be more appropriate in the present case as the troposphere has been divided into three layers. Obviously each of these layers varies in the concentration of the pollutants and hence chemical reaction will take place with different rate in these layers. The model is supported by appropriate boundary and initial conditions.

INTRODUCTION

The diffusion process in the atmosphere from a point area source has been studied in various contexts such as air pollution [2] – [6] and pheromone diffusion of biological organism in the atmosphere [8]. In particular, the dispersion of an air pollutant from a point source has been investigated by many workers. During dispersion in the atmosphere the primary species is sometimes converted to secondary species and both primary and secondary species may be removed by rainout/washout and the like. Alam and Seinfeld [1] studied the dispersion of sulferdioxide and sulfate in the atmosphere by solving steady state three dimensional diffusion equations and discussed the effects of the conversion and removal processes by considering these as first order process.

On the other hand at the industrial agriculture site, where the sources of pollutant emission are two dimensional, it may be possible that under certain circumstances the emittence of pollutants from these sources may be non steady and thus it affects the reaction rate which varies at different heights in the atmosphere [9]. For example, in atmosphere the concentration of species such as NO_2 and SO_2 varies extensively with respect to time. Thus, study of transient problems would be more appropriate in dealing with such cases. In the following work an attempt has been made to investigate transient state dispersion of pollutants.

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MATHEMATICAL FORMULATION

A mathematical equation describing the time averaged concentration of chemically reactive species with wind velocity and pollutant generation is given as :

$$\frac{\partial C}{\partial t} + U\,\frac{\partial C}{\partial x} = \frac{\partial}{\partial z}\left(D\frac{\partial C}{\partial z}\right) + Q + R \qquad ...(1)$$

where z is vertical distance from the ground ($z = 0$).

For the steady atmosphere condition *i.e.* when there is no wind, equation (1) reduces to

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial z}\left(D\frac{\partial C}{\partial z}\right) + Q + R \qquad ...(2)$$

where C is the pollutant concentration in the air, D, Q and R are eddy diffusivity, pollutant generation rate and rate of chemical reaction respectively.

The mathematical description of the model (2) is completed by the following initial and boundary conditions.

Initially, it is assumed that concentration of pollutants in the atmosphere is negligible and therefore, the initial concentration is taken as

$$C = 0 \quad \text{at} \quad t = 0 \qquad ...(3)$$

At the height $z = z_0$, pollution concentration is prescribed as

$$C = C_0 \quad \text{at} \quad z = z_0 \qquad ...(4)$$

where, z_0 is the vertical distance from datum. The pollutant concentration approaches zero far from the source.

$$C = 0 \text{ at large } z \qquad ... (5)$$

SOLUTION

Equation (2) on comparing with Euler Lagranges equation gives the following variational form for i layers.

$$I_i = \frac{1}{2}\int_{z_{i-1}}^{z_i}\left[D^{(i)}\frac{\partial C^{(i)2}}{\partial_z} - 2Q^{(i)}C^{(i)} - 2R^{(i)}C^{(i)} + \frac{\partial C^{(i)2}}{\partial t}\right]dz \quad ...(6)$$

The lower atmosphere (troposphere) is divided into three layers. The generation is assumed to be taking place at constant rate in different layers. In light of descritization of lower atmosphere, the layer wise description of parameters is given as follows :

First Layer : $0 \le z \le z_1$

$$D^{(i)} = D_1,\ Q^{(i)} = Q_1,\ R^{(i)} = R_1 \quad ...(7)$$

Second Layer : $z_1 \le z \le z_2$

$$D^{(i)} = D_2,\ Q^{(i)} = Q_2,\ R^{(i)} = R_2 \quad ...(8)$$

Third Layer : $z_2 \le z \le z_3$

$$D^{(i)} = D_3,\ Q^{(i)} = Q_3,\ R^{(i)} = R_3 \quad ...(9)$$

Here we are assuming that the pollutant concentration $C^{(e)}$ (say) form e^{th} layer, varies linearly within the layer, however it has definite unknown value C_2.

$$C^{(e)} = \begin{Bmatrix} a^{(e)} + b^{(e)}z, & z_i < z < z_{i+1} \\ c_i, & z = z_i \end{Bmatrix} \quad ...(10$$

for $i = 0, 1, 2$

Equation (1) can be put in matrix form as

$$C^{(e)} = P^T N^{(e)} C^{(e)} \quad ... (11)$$

where, $P^T = [1\ z]$

Expressing the functional I in (6) symbolically as follows :

$$N^{(e)} = \begin{pmatrix} 1 & z_1 \\ 1 & z_{i+1} \end{pmatrix}^{-1} \quad \text{and} \quad C^{(e)} = \begin{pmatrix} C_1 \\ C_{i+1} \end{pmatrix}$$

Expressing the functional I in (6) symbolically as follows :

$$I_i = I_d^{(i)} - I_k^{(i)} + I_C^{(i)} \quad ...(12)$$

where,

$$I_d^{(i)} = \frac{1}{2}\int_{z_{i-1}}^{z_1} D^{(i)}\left(\frac{\partial C^{(i)2}}{\partial z}\right)dz \quad ...(13)$$

$$I_k^{(i)} = \int_{z_{i-1}}^{z_i} (Z^{(i)} + R^{(i)})\, C^{(i)}dz \quad ...(14)$$

$$I_C^{(i)} = \frac{1}{2}\int_{z_{i-1}}^{z_i} D^{(i)}\left(\frac{\partial C^{(i)2}}{\partial z}\right)dz \quad ...(15)$$

Now use the parameter values from equation (7), (8) and (9) and interpolating function defined in (10) into (13), (14) and (15) in order to evaluate these integrals and hence I_i in equation (12). The integrals I_i are then assembled to obtain

$$I = \sum_{i=1}^{3} I_i$$

Following Ritz method to extemize I with respect to C_i by putting

$$\frac{dI}{dC} = 0 \quad ...(16)$$

$$U\frac{dC}{dt}K_1C = K_2 \quad ... (17)$$

Where, $C = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$

K_1 and U are the matrices of order 2 × 2 and K_2 is a vector of order 2 × 1, whose values for e^{th} element is given below :

$$K_1^{(e)} = \frac{D^{(e)}}{(Z_j - Z_i)}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$K_2^{(e)} = \frac{Q^{(e)} + R^{(e)}}{(Z_j - Z_i)}\begin{pmatrix} z_i \\ z_i - 2z_i \end{pmatrix}$$

and $$U^{(e)} = \frac{Z_j - Z_i}{6}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

Solving the system (17) analytically, employing the method of Laplace transform, to obtain the concentration C_i, $i = 1, 2$

$$UC + K_1C = K_2t \quad ...(18)$$

NUMERICAL RESULTS AND DISCUSSION

The equation (18) is computed numerically for concentration distribution using the parameter values given in the table below :

Table 1.

Layer	D (Km^2/sec.)	Q (Kg./sec.)	R (Km^3/gm/sec.)	z (Km)
I	2×10^{-5}	5×10^{-3}	2×10^{-12}	5
II	3×10^{-5}	5×10^{-3}	4×10^{-12}	10
III	2.5×10^{-5}	0	6×10^{-12}	15

In this paper the concentration of the pollutants has been computed at different heights in the atmosphere (troposphere), in particular for different time. The results are drawn graphically in Fig.1. The results are compatible with

the boundary conditions as is shown in Fig.1. The concentration at the highest level *i.e.*, h = 15 km. becomes zero and as we move down wards it increases gradually and becomes maximum at the surface (known concentration, 90 mgms.). At t = 0 Ci's (i = 1,2) are zero due to initial condition Fig.2. It is observed from the graph that there is gradual increase in the pollutant concentration upto the height of 5 km for time 30, 60, 90 and 120 min. After this height the concentration becomes steady.

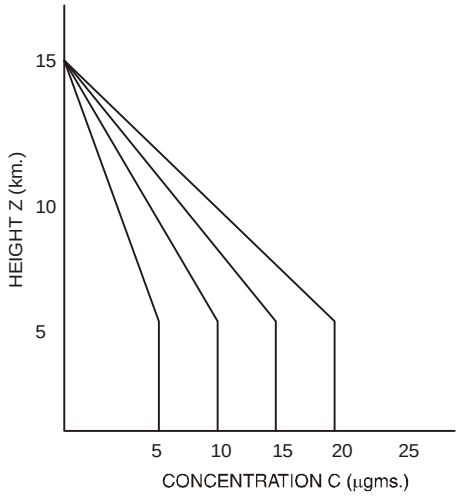

Fig.1.

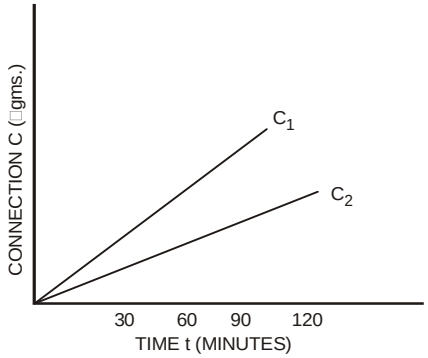

FIg.2.

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